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LETTERS TO THE EDITORS

COMMENTS ON ACTIVATION OF NUCLEATION CAVITIES ON A HEATING SURFACE WITH TEMPERATURE GRADIENT IN SUPERHEATED LIQUID

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MADEJSKI [1] has presented an interesting analysis on the activation of nucleation cavities in the presence of a non-uniform temperature field. In contrast to previous treatments of the subject, e.g. Hsu [2], whose analysis apparently provided the motivation for Madejski's work, the active bubble nucleus is allowed to take the shape of a flattened spheroid. It is shown that the active bubble nucleus is spherical only for the case of uniform superheat and that if a temperature gradient exists in the surrounding liquid, the shape of the bubble nucleus is flattened. As a consequence, the superheat required for activation is greater than in the case of uniform superheat. As with Hsu's analysis, the solution is given in terms of a thermal layer thickness, however, as the thermal layer thickness approaches infinity, Madejski's expression does reduce to the uniform superheat expression as would be required. Unfortunately, *accurate* data involving known cavity radii, thermal layer thickness and wall superheat are not yet available in the published literature to test the analysis.

Since such data will probably be available in the near future, it seems advisable to point out an error in the analysis. The notation and equation numbers which follow are those used by Madejski.

The definition for k in equation (27) of Madejski's paper is incorrect and does not yield either equation (28) or (30). In order to find the correct expression, it is convenient to redefine the variable t in the following manner

$$\frac{\gamma}{R_c}\left(y - \frac{y^2}{2\delta}\right) = t^2 - 1 = \frac{\gamma}{R_c}\int_0^y \theta \, dy. \tag{23}$$

Thus using the same procedure as suggested by Madejski, equation (25) becomes

$$\gamma = -2\int_1^{\sqrt{2}} \frac{(1 - t^2)\, dt}{\sqrt{[2 - t^2]}\sqrt{\left[1 - \frac{2R_c}{\delta\gamma}(t^2 - 1)\right]}} \cdot \tag{25}$$

Using the substitution

$$t = \sqrt{2}\sin\psi \tag{26}$$

and introducing the quantity

$$k = \sqrt{2}\left(\frac{\delta\gamma}{2R_c} + 1\right)^{-\frac{1}{2}} \tag{27}$$

as suggested by the scheme presented in [3], equation (25) is reduced to the form of elliptic integrals of the first and second kind, defined by

$$F(k, \psi) = \int_0^\psi \frac{d\psi}{\sqrt{[1 - k^2 \sin^2\psi]}}$$

$$E(k, \psi) = \int_0^\psi \sqrt{[1 - k^2\sin^2\psi]}\, d\psi \tag{29}$$

thus we obtain

$$\gamma = 2\left(1 - \frac{k^2}{2}\right)^{\frac{1}{2}}\left\{\left(\frac{2}{k^2} - 1\right)\left[F\left(k, \frac{\Pi}{2}\right) - F\left(k, \frac{\Pi}{4}\right)\right] - \frac{2}{k^2}\left[E\left(k, \frac{\Pi}{2}\right) - E\left(k, \frac{\Pi}{4}\right)\right]\right\}. \tag{30}$$

It is important to note that the form of equation (30) remains unchanged; only the multiplying factor differing from the original analysis.

Proceeding further, substituting equation (27) into Madejski's equation (22)

$$b = \delta\left\{1 - \sqrt{\left[\frac{2(1 - k^2)}{2 - k^2}\right]}\right\} \tag{31}$$

from which

$$1 + \left(\frac{R_1}{R_2}\right)_m = 1 + \frac{1}{2}\left(1 + \frac{b^2}{R_c^2}\right)$$

$$= 1 + \frac{1}{2}\left\{1 + \left(\frac{\delta}{R_c}\right)^2\left(1 - \sqrt{\left[\frac{2(1 - k^2)}{2 - k^2}\right]}\right)^2\right\} \tag{32}$$

and thus as indicated by Madejski, $k = 0$ yields $b = 0$, and $k = 1$ yields $b = \delta$. Thus

$$0 \leqslant k \leqslant 1.$$

Substituting equation (27) into equation (30), an expression is obtained for the dimensionless temperature gradient as a function of the parameter k.

$$\frac{R_c}{\delta} = \frac{1}{\sqrt{[2(2-k^2)]}}\left\{(2-k^2)\left[F\left(k,\frac{\Pi}{2}\right) - F\left(k,\frac{\Pi}{4}\right)\right] - 2\left[E\left(k,\frac{\Pi}{2}\right) - E\left(k,\frac{\Pi}{4}\right)\right]\right\}. \qquad (33)$$

This corrected relationship is shown in Fig. 1 as the solid line.

Using Madejski's equation (14) along with equation (27),

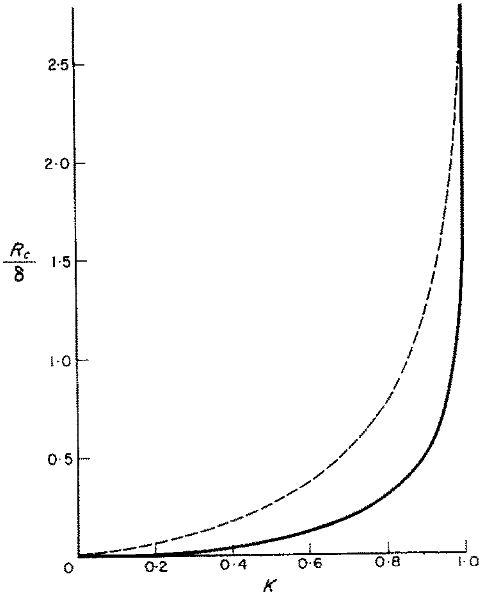

FIG. 1. Variation of dimensionless temperature gradient with parameter k.

(32) and (33) one obtains an expression for the ratio of the superheat required for activation in non-uniform temperature field to that required in a uniform field.

$$\frac{\rho'-\rho''}{\rho'}\frac{R_c P_s' \Delta T}{2\sigma} = \frac{\Delta T}{\Delta T_\infty} = \frac{(2-k^2)}{k^2}\frac{R_c}{\delta}\left\{1 + \frac{1}{2}\left[\left[1 + \frac{\delta^2}{R_c^2}\left(1 - \sqrt{\left[\frac{2(1-k^2)}{2-k^2}\right]}\right)^2\right]\right]\right\}.$$

As the dimensionless temperature gradient (R_c/δ) approaches zero, i.e. uniform superheat, $\Delta T/\Delta T_\infty$ approaches unity. As R_c/δ becomes large, we have the asymptotic solution

$$\frac{\Delta T}{\Delta T_\infty} = \frac{3}{2}\frac{R_c}{\delta}.$$

Combining equations (31) and (33), an expression is obtained for the ratio of the height of the bubble nucleus to the cavity radius

$$\frac{b}{R_c} = \frac{\{1 - \sqrt{[2(1-k^2)/(2-k^2)]}\}}{R_c/\delta}$$

the corrected expressions for $\Delta T/\Delta T_\infty$, and b/R_c are shown as the solid lines on Fig. 2.

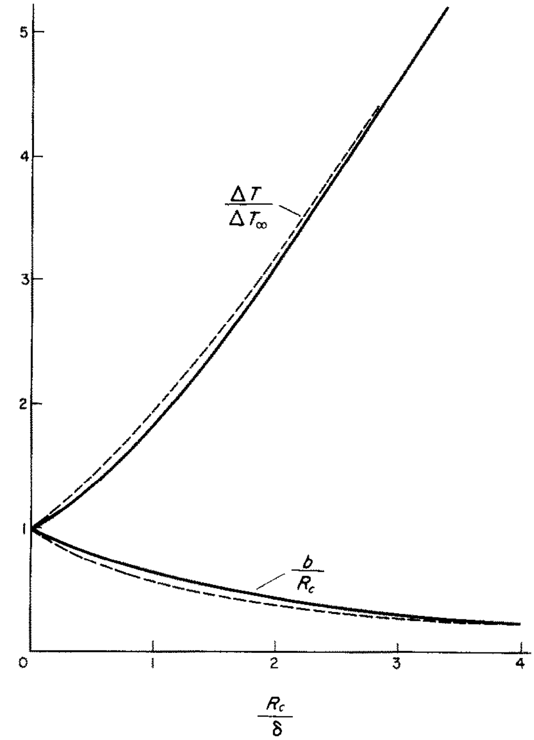

FIG. 2. Superheat and bubble geometry ratios as a function of dimensionless temperature gradient.

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FURTHER REMARKS ON A MODEL OF TWO-DIMENSIONAL CONVECTION

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IN MY recent article [1] I demonstrated that a certain model of steady two-dimensional convection at high Rayleigh number fails to describe either the motion of a fluid with infinite Prandtl number or the motion of a fluid in a porous medium. G. Roberts of the University of Newcastle-upon-Tyne has pointed out to me that there is an unstated assumption in that model, that the temperature is of unit order-of-magnitude in the vertical boundary layers. If this assumption is replaced by a fifth model balance that $(\vec{u}.\vec{\nabla})\theta \sim \nabla^2\theta$ in that region, allowing $[\theta]$ to be less than unity in order of magnitude, self-consistent models may be constructed.

The balances in the infinite Prandtl number fluid (obtained by G. O. Roberts [27]) are $[u_{int}] = Ra^{3/5}$, $\delta_H = Ra^{-1/5}$, $\delta_v = Ra^{-3/10}$, $[\theta] = Ra^{-1/10}$, $Nu \sim Ra^{1/5}$ (where $[\theta]$ is the order of magnitude of the temperature in the vertical layers), and for motion in a porous medium $[u_{int}] = A^{2/5}$, $\delta_H = A^{1/5}$, $\delta_v = A^{-2/5}$, $[\theta] = A^{-1/5}$, $Nu \sim A^{1/5}$.

The balances for motion of a viscous fluid with large Prandtl number become $[u_{int}] = Ra^{2/3}Pr^{-1/9}$, $\delta_T = Ra^{-1/3}Pr^{2/9}$, $\delta_H = Ra^{-1/3}Pr^{5/9}$, $\delta_v = Ra^{-1/3}Pr^{1/18}$, $[\theta] = Pr^{-1/6}$, $Nu \sim Ra^{1/3}Pr^{-2/9}$ for the Robinson model and $[u_{int}] = Ra^{1/2}Pr^{-1/6}$, $\delta_T = Ra^{-1/4}Pr^{1/12}$, $\delta_H = Ra^{-1/4}Pr^{5/12}$, $\delta_v = Ra^{-1/4}Pr^{-1/12}$, $[\theta] = Pr^{-1/6}$, $Nu \sim Ra^{1/4}Pr^{-1/12}$ for the Pillow model. The conclusion that the viscous boundary layer eventually fills the cell remains valid; it is however now seen that the temperature in the vertical boundary layers decreases in order of magnitude as the Pradntl number increases. The finite Prandtl number model is valid for $Pr \ll Ra^{3/5}$ for both models.

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